Solve the following differential equations. **SCORE: _____ / 30 PTS**

[a] $6xy\,dx + (4y + 9x^2)\,dy = 0$ 11 PTS **FINAL ANSWER:** $3x^2y^3 + y^4 = C$

[b] $(1+x)\dfrac{dy}{dx} - xy = x + x^2$ 9 PTS **FINAL ANSWER:** $y = -x - 2 + \dfrac{Ce^x - 1}{1+x}$

[c] $(x^2 + y^2)\,dx + (x^2 - xy)\,dy = 0$ 10 PTS **FINAL ANSWER:** $(x+y)^2 = Cxe^{\frac{y}{x}}$

[a] $\underbrace{6xy}_{M}\,dx + \underbrace{(4y + 9x^2)}_{N}\,dy = 0$

$$\frac{N_x - M_y}{M} = \frac{18x - 6x}{6xy} = \frac{12x}{6xy} = \boxed{\frac{2}{y}}$$

$$\mu = e^{\int \frac{2}{y}\,dy} = e^{2\ln|y|} = y^2$$

$$\underbrace{6xy^3}_{M}\,dx + \underbrace{(4y^3 + 9x^2y^2)}_{N}\,dy = 0$$

CHECKPOINT: $M_y = 18xy^2 = N_x$ ✓

$$f = \int 6xy^3\,dx$$

$y^4 + 3x^2y^3 + C(x)$ OR $= 3x^2y^3 + C(y)$

$$f_y = 9x^2y^2 + C'(y) = 4y^3 + 9x^2y^2$$

$6xy^3 + C'(x) = 6xy^3$ OR $C'(y) = 4y^3$

$C(x) = 0$ OR $C(y) = y^4$

$$f = 3x^2y^3 + y^4 = C$$

[b] $\dfrac{dy}{dx} - \dfrac{x}{1+x}\,y = x$

$$\mu = e^{\int -\frac{x}{1+x}\,dx} = e^{\int (-1 + \frac{1}{1+x})\,dx}$$

$$= e^{-x + \ln|1+x|} = (1+x)e^{-x}$$

$$(1+x)e^{-x}\frac{dy}{dx} - xe^{-x}y = (x + x^2)e^{-x}$$

CHECKPOINT: $\dfrac{d}{dx}(1+x)e^{-x}$

$$= e^{-x} - (1+x)e^{-x}$$

$$= -xe^{-x} \checkmark$$

(3) $(1+x)e^{-x}y = -(3 + 3x + x^2)e^{-x} + C$

$$y = -\left(x + 2 + \frac{1}{1+x}\right) + \frac{Ce^x}{1+x}$$

$$= -x - 2 + \frac{Ce^x - 1}{1+x}$$

u		dv
$x + x^2$	+	e^{-x}
$1 + 2x$	−	$-e^{-x}$
2	+	e^{-x}
0		$-e^{-x}$

$$(-x - x^2 - 1 - 2x - 2)e^{-x}$$

$$= -(3 + 3x + x^2)e^{-x}$$

EACH UNDERLINED ITEM (OR DESIGNATED ALTERNATE) IS WORTH 1 POINT UNLESS INDICATED OTHERWISE

[c] $y = vx$

$(x^2 + v^2x^2)\,dx + (x^2 - vx^2)(v\,dx + x\,dv) = 0$

$(x^2 + vx^2)\,dx + (x^3 - vx^3)\,dv = 0$

$x^2(1+v)\,dx + x^3(1-v)\,dv = 0$

② $\int \frac{1-v}{1+v}\,dv = \int -\frac{1}{x}\,dx$

$\int \left(-1 + \frac{2}{1+v}\right) dv = -\ln|x| + C$

$-v + 2\ln|1+v| = -\ln|x| + C$

$\frac{(1+v)^2}{e^v} = \frac{C}{x}$

$\frac{\left(1+\frac{y}{x}\right)^2}{e^{\frac{y}{x}}} = \frac{C}{x}$

$\frac{(x+y)^2}{e^{\frac{y}{x}}} = Cx$

$(x+y)^2 = Cxe^{\frac{y}{x}}$